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Peter Flaschel, Florian Hartmann,
Chris Malikane and Christian R. Prosano

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INSTITUTE OF EMPIRICAL ECONOMIC RESEARCH
Osnabrück University
Rolandstraße 8
49069 Osnabrück
Germany

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Peter Flaschel
Bielefeld University, Bielefeld, Germany

Florian Hartmann*
University of Osnabrück, Osnabrück, Germany

Chris Malikane
University of the Witwatersrand, Johannesburg, South Africa

Christian R. Proaño
The New School for Social Research, New York

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Abstract

The paper investigates the emergence of complex market expectations (opinion dynamics) around nominal exchange rate adjustments using a macro-financial model of a small open economy featuring heterogeneous expectation formation (chartists and fundamentalists) and gradual adjustment processes in real and also to a certain degree in financial markets. The model shows among other things the mechanisms through which the first type of agents tends to destabilise the economy. Global stability can be ensured if opinions turn to fundamentalist behaviour far off the steady state. This interaction of expectations and population dynamics is bounding the – due to chartist behavior – potentially explosive real-financial market interactions, but can enforce irregular behaviour within these bounds. The size of output and exchange rate fluctuations can be dampened by adding suitable policy measures to the dynamics of the private sector.

Keywords: Nonlinear Exchange Rate Dynamics, Opinion Dynamics, Viability, Persistent and Irregular Fluctuations, Macroeconomic Policy

JEL classifications: E12, E24, E31, E52.

*Corresponding author. Email: florian.hartmann@uni-osnabrueck.de

1 Introduction

As discussed by Kindleberger and Aliber (2005), the evolution of the world economy has been characterized by recurrent financial and exchange rate crises which, due to their often devastating macroeconomic and social effects, have raised important issues for theorists and policy makers. In this context, the ruling paradigm in macroeconomics of Dynamic Stochastic General Equilibrium (henceforth, DSGE) modelling has done a rather non-convincing job in explaining the recurrent character of such financial phenomena and especially the recent global downturn as argued by Colander et al. (2009), and also admitted by some of its proponents such as Chari et al. (2009). Arguably, this unsatisfactory performance has not been the result of a lack of mathematical sophistication. Rather, it derives from the adoption of an equilibrium approach coupled with the assumption of Rational Expectations, which jointly seem questionable both from the methodological and the empirical perspective.

In the behavioral finance literature – in contrast to the predominant macroeconomic literature – it is widely acknowledged that the rational expectations assumption is not able to explain basic stylized facts of financial markets – not only concerning crises –, see e.g. Hommes (2006) and De Grauwe and Grimaldi (2006), and alternative expectations formation schemes are widely used.

This paper proposes a number of departures from DSGE methodology, which can be seen as the building blocks of a new approach in the Keynesian tradition[1], by constructing a macrodynamic model along the lines of Dornbusch (1976), which incorporates basic important feedback channels between the real and the financial sector, and in which markets are not assumed to jump to their equilibrium positions, but where dynamic adjustment processes are taking place. One of the key contributions of the paper is the explicit incorporation of market expectations or opinion dynamics in financial markets populated by heterogeneous agents. This allows us to examine the effects of herding and speculative behaviour in a simple macrodynamic framework of a small open economy as proposed by Franke (2011). More precisely, we adopt the distinction between *chartists* and *fundamentalists* widely used in the literature on exchange rate dynamics, see e.g. De Grauwe and Grimaldi (2005), Proaño (2011) and Chiarella et al. (2011). Chartists behave like speculators and can be seen as technical traders who adopt a simple adaptive expectation mechanism. In contrast, fundamentalists focus on basic economic data and expect variables to return to steady state values with a certain adjustment speed. Chartists tend to exert a destabilising influence on the economy, whereas the presence of fundamentalists is generally stabilising.

Albeit simple, this description of agent heterogeneity on financial markets is consistent with empirical studies that analyse expectational heterogeneity, such as Frankel and Froot (1990) and Menkhoff et al. 2009). This description is also sufficient to examine some of

[1]Further contributions to this line of research in DSGD modelling are e.g. Charpe et al. (2011, 2012a and 2012b)

the core features of financial markets that have played a prominent role in the recent global financial crisis. Overall *market* expectations are here a function of individual fundamentalist and chartist expectations, and depend on the relative weight of each group in the market. Hence, the model economy contains two potential sources of instability: the feedbacks between real and foreign exchange markets via the nominal exchange rate e, and the endogenous market expectations dynamics produced by the interaction of heterogenous agents on asset markets. Thus, it allows us to investigate a key question emerging from the current financial crisis, namely whether unfettered, interconnected markets with heterogeneous agents are able to absorb external shocks, or rather tend to amplify them.

We prove that the resulting 4D dynamical system describing the evolution of the economy always has either a single steady state characterised by uniformly distributed agents, or three steady states (one with uniformly distributed agents, a chartist and a fundamentalist one). Even though various subdynamics of the model can be stable (at the uniform or fundamentalist steady state), the complete system may be repelling around all of its equilibria. Given the complexity of the 4D nonlinear system, it is difficult to draw more precise conclusions on the overall dynamics (up to the consideration of two supplementing 2D cases). Therefore we adopt numerical methods to further explore the dynamic properties of the model. The numerical simulations show that the 4D system is indeed *viable*: all trajectories remain in an economically meaningful subset of the state space. In this sense, the model shows the somewhat surprising result that unfettered markets with possibly accelerating real-financial feedback mechanisms have some in-built stabilising mechanism (based on opinion dynamics) that prevent the economy to move on an infeasible path. Moreover, despite the trivial dynamics of the 2D subsystems, the full 4D dynamics can exhibit irregular and even complex motions. Hence, if all of the steady states are repelling, the system can exhibit irregular, though persistent real-financial market fluctuations. The considered opinion dynamics is therefore capable of ensuring upper and lower turning points in the real-financial market interactions, but the generated persistent fluctuations may still be too large to be acceptable from the societal point of view.

The remainder of the paper is organized as follows. In the next section we briefly discuss the Dornbusch (1976) exchange rate dynamics in order to motivate the subsequent framework as well as to highlight its main features. In section 3 we then extend the Dornbusch framework through the incorporation of heterogeneous behavioral expectations and endogenous market expectations formation. We analyze the resulting framework by means of numerical simulations in section 4, focusing on the stability of the system, and in section 5 possible policy measures to tame fluctuations are reviewed. Section 6 draws some concluding remarks.

2 The Dornbusch Exchange-Rate Dynamics under Rational Expectations

Since Dornbusch's (1976) approach to exchange rate dynamics serves as a point of departure of the model put forth in this paper, we firstly recapitulate its basic properties. The Dornbusch (1976) exchange rate dynamics reads in the case of the Uncovered Interest Parity condition (UIP) with myopic perfect foresight on the exchange rate dynamic in their simplest form as follows (with e, p, y the logarithms of the exchange rate, the price and the output level, the foreign price level being normalized to 1):

$$\begin{aligned} \dot{p} &= \beta_p(y^d(e-p)-\bar{y}) \\ \dot{e} &= i(p)-\bar{\imath}^* \end{aligned}$$

We assume that the economy is at its full employment level $\bar{y}$, but that deviations of aggregate demand y^d (which only depend on the real exchange rate here) from this level determine with adjustment speed β_p the rate of inflation. The second law of motion is the UIP condition, under the assumption of myopic perfect foresight. The relationship $i(p)$ is a standard inverted LM curve, i.e., it describes a positive relationship between the price level and the domestic nominal rate of interest. Clearly the steady state is of saddle-point type (the determinant of the Jacobian is negative) and the implied phase diagram is shown in figure 1.

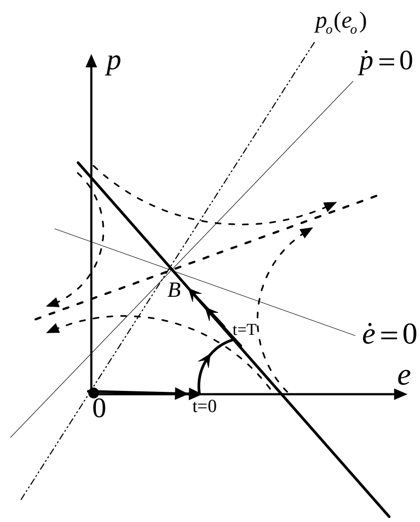

Figure 1: The Dornbusch exchange rate dynamics under myopic perfect foresight

The rational expectations school solves such a dynamical system in the following way. It assumes that the economy is (if no anticipated shocks are occurring) always on the stable manifold of the given saddle-point, horizontally to the right of the old steady state 0, in the intersection of the stable manifold (a straight line in this simple model) with the horizontal axis, if an unanticipated shock that moves the steady state to point B has occurred, since the price level can only adjust gradually in this model. We therefore get an overshooting exchange rate (with respect to its new steady state values) and thus an increase in goods demand, which increases the price level and the nominal interest rate gradually. This then appreciates the exchange rate from its excessively high level until all variables reach the new steady state.

In the case of anticipated shocks the situation that is assumed by the rational expectations school becomes more complicated, since at the time of the announcement of the policy we are still in the old phase diagram around the point 0. In this case, the exchange rate jumps at $t = 0$ to the right to a uniquely determined level. From there it uses the unstable saddle-point bubble in the old dynamics, which starts at this point. The exchange rate then reaches the stable manifold of the new dynamics exactly at the time T when the announced policy shock actually takes place. We thus have – depending on T – a jump in the exchange rate that may still overshoot its new steady state position and which then switches immediately towards a bubble of length T with both rising prices and exchange rates, from which it departs through a soft landing on the new stable manifold at time T.

This is – when appropriately extended – the rational expectations approach to exchange rate dynamics in the frame of a Dornbusch IS-LM model with a price Phillips curve. Its solution techniques looks attractive, since it provides – in addition to the usual treatments of shocks – also a well-defined answer in the case of anticipated events (within certain bounds, depending on the size of the anticipated shock). However, the rational expectations approach may also be viewed as a rather heroic solution to the treatment of (un)anticipated demand, supply and policy shocks from a descriptive point of view.

3 A Macrodynamic Framework Applying Franke's (2011) Endogenous Market Expectations Formation

3.1 The Dornbusch model with somewhat sluggish exchange rate adjustments

As previously mentioned the main purpose of this paper is to analyse the specific sources of macroeconomic instability induced by FX markets characterized by an endogenous market expectations formation. That is the reason why we focus on the Dornbusch and opinion dynamics part and neglect other sources of instability. Following this modelling strategy, we simplify the real part by ignoring inflation and growth, and by representing the quantity adjustment process by means of a dynamic multiplier approach. This simplifies the Metzlerian inventory accelerator mechanism of the real-side oriented Keynes-Metzler-Goodwin model of

Chiarella and Flaschel (2000), thus suppressing it as a source of instability.[2] As a result, the real part of the economy is always stable (from this partial perspective), provided the propensity to spend is less than one. However, we assume that FX markets have real effects on investment and consumption.

To be precise, we assume that output moves according to a standard dynamic multiplier process.[3]

$$\dot{Y} = \beta_y(Y^d - Y) = \beta_y((a_y - 1)(Y - Y_o) - a_i(i - i^*) + a_e(e - e_o), \tag{1}$$

where Y_o is the given steady state level of output, a_y the propensity to spend, a_e the impact of the exchange rate e on aggregate demand, a_i the impact of the interest rate i on aggregate demand and where β_y is the speed of adjustment concerning goods-market disequilibria. For the sake of concreteness we measure e by AUD/USD with Australia as the domestic economy.

There is only one risky asset traded by the agents of the domestic economy, foreign bonds: B^*, which are subject to exchange rate risk. Following Chiarella et al. (2009), we postulate a dynamic disequilibrium adjustment process for the nominal exchange rate e :

$$\hat{e} = \beta_e \alpha_e \sigma_e(i^* + \pi_e^e - i), \quad \alpha_e \in (0, 1). \tag{2}$$

In words, only a fraction α_e of current aggregate excess demand for the foreign bonds *stock* $\sigma_e(\cdot)$ actually enters the foreign exchange market owing to the existence of adjustment costs. Thus, $1/\alpha_e$ represents the delay with which agents wish to clear any stock imbalance $\sigma_e(\cdot)$. As Chiarella et al. (2009) have argued, this approach is necessary in an open economy in a continuous time framework where *flow* rather than stock imbalances must enter the capital account of the balance of payments. The flow processes on asset markets are then translated into asset price changes by the speed of adjustment parameter β_e.

In addition to B^* (with USD price 1) we have in the Australian economy domestic short-term fix-price bonds, B with AUD price 1. The central bank is assumed to fix at each moment of time the interest rate on B at the level i according to the rule

$$i = i^* + i_y(Y - Y_o) + i_e(e - e_o), \quad i_y, i_e > 0, \tag{3}$$

with i^* the given foreign rate of interest. Equations (1)-(2) represent our baseline model of the real financial interaction in a small open economy which is kept as simple as possible since we want to highlight the interaction of this part with the dynamics of expectations formation.

In order to investigate the stability characteristics of the real-financial interaction, assume for the time being that capital gain expectations π_e^e, occurring when there is currency depreciation and switching back into domestic bonds, are given and zero. We then simply have

[2] The instability induced in the KMG approach by the wage-price spiral as discussed e.g. by Flaschel and Krolzig (2006) is also ignored.

[3] For any dynamic variable x, $\dot{x}$ denotes its time derivative, $\widehat{x}$ denotes its rate of growth, and x_o denotes its steady state value.

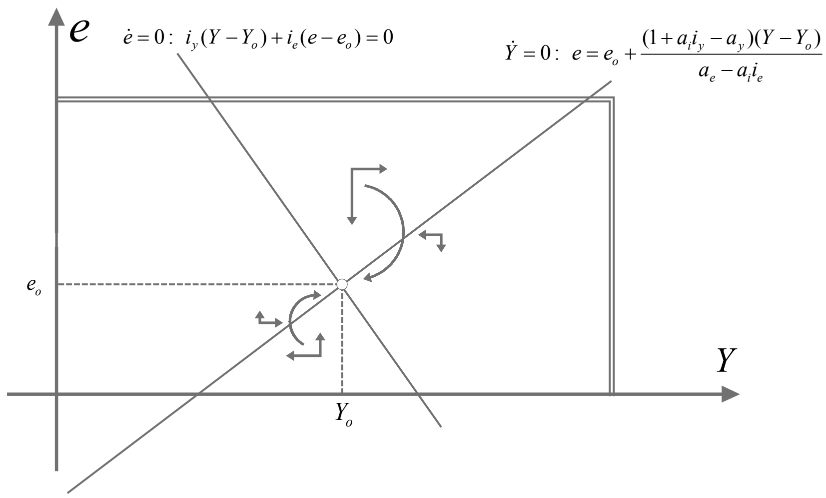

Figure 2: Asymptotically stable real-financial market interaction.

for the matrix of partial derivatives evaluated at the steady state, the Jacobian J of the real-financial market interaction, when the Taylor rule is inserted into the output and the Dornbusch exchange rate dynamics:

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} - & + \\ - & - \end{pmatrix}$$

if the parameter i_e is chosen sufficiently small. i_e is a policy parameter chosen by the domestic central bank and, therefore, can be expected to be carefully adjusted due to exchange rate targeting purposes. The sign structure resulting on this prerequisite gives for the Jacobian matrix trace $trJ < 0$ and determinant $detJ > 0$ and, hence, implies according to Routh-Hurwitz conditions for two-dimensional continuous-time dynamic systems two eigenvalues with negative real parts.

The real-financial interaction with stationary expectations is thus asymptotically stable around the steady state. This case is illustrated in Figure 2.

These conclusions only concern the interaction of real and financial adjustment processes and do not depend on the presence of behavioural traders on the financial markets, which are introduced in the next section.

3.2 Behavioral expectations

We consider financial markets with heterogeneous agents in the DSGD modelling approach and, following the behavioral approach to macroeconomic dynamics, employed e.g. by Chiarella et al. (2008) and mentioned by Brunnermeier (2008). Traders of foreign bonds are distinguished as *fundamentalists*, f, and *chartists*, c. Fundamentalists expect capital gains in the foreign exchange market to converge back to their steady state position (zero in our model) with speed $\beta_{\pi^e_{ef}}$. Chartists instead adopt a simple adaptive mechanism to forecast the evolution of capital gains in FX markets $\dot{\pi}^e_e$. The adoption is transmitted with speed $\beta_{\pi^e_{ec}}$ to the chartists' expectation formation. Formally:

$$\begin{aligned} \dot{\pi}^e_{ef} &= \beta_{\pi^e_{ef}}(0 - \pi^e_{ef}), \\ \dot{\pi}^e_{ec} &= \beta_{\pi^e_{ec}}(\hat{e} - \pi^e_{ec}). \end{aligned}$$

To be sure, more complex expectation formation mechanisms can be adopted for each type of agent, including forward looking rules, in particular if numerical simulations are intended.[4] Yet, our formulation has the virtue of analytical simplicity, and it allows us to draw a sharp distinction with respect to Rational Expectation models.

Given that agents have heterogeneous expectations, it is not obvious a priori what the *market* expectations should be. In standard equilibrium models with efficient markets, heterogeneous information and beliefs are spontaneously aggregated and made uniform under the pressure of market forces. This is clearly not the case in our framework. As a first step, suppose that the population shares of chartists and fundamentalists, ν_c and $(1 - \nu_c)$, respectively, are constant.[5] It may be tempting to argue that market expectation is the weighted average of the expectations of chartists and fundamentalists:

$$\pi^e_e = \nu_c \pi^e_{ec} + (1 - \nu_c)\pi^e_{ef}.$$

It is not clear, however, that this is a theoretically appropriate way of capturing the formation of market expectations. For market expectations π^e_e may actually reflect what both types of agents *think* will emerge from the process of aggregation of fundamentalist and chartist expectations. In other words, market expectations may reflect the agents' view about the "average" opinion. And this need not be the exact, weighted average of the individual expectations. In turn, the law of motion of market expectations may be the product of what on average agents think the average opinion and its rate of change will be. In the words of Keynes (1936, p.156):

> *It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.*

[4] E.g., Assenza et al. (2012) investigate systematically dynamic implications of various sets of expectation formation schemes for a macroeconomic equilibrium model. A similar procedure to analyse rigorously consequences of different expectational mechanisms could be applied to our model.

[5] Population shares are endogenised in the next section.

In this paper, we consider the following simple law of motion for aggregate capital gain expectations:

$$\dot{\pi}_e^e = \beta_{\pi_e^e}[\nu_c \hat{e}(\cdot) - \pi_e^e], \tag{4}$$

where $\beta_{\pi_e^e} > 0$ represents an adjustment speed parameter and where the nominal exchange rate depreciation only enters expectations with the weight ν_c of the chartists (since the change in their number is not foreseen). We thus assume that adaptive expectations formation drives the expectation of capital gains (to the extent chartists are present in the market), while fundamentalists are only adding stabilizing forces to it. To be sure, this is only one possible formalisation of the dynamics of aggregate expectations in markets with heterogeneous agents, and alternative approaches can be proposed (see, for example, the approach adopted by De Grauwe and Grimaldi (2005) in their analysis of the behaviour of agents on foreign exchange markets). Yet, we regard equation (4) as a very parsimonious way of capturing *both* the influence of aggregate observed variables *and* the role of heterogeneity and self-driving forces in expectation formation.

In order to analyse the dynamics of this economy, note that if the weight of chartists in average expectation is zero, the Jacobian of the 3D system (1), (2), (4) at the steady state becomes

$$J = \begin{pmatrix} - & + & 0 \\ - & - & + \\ 0 & 0 & - \end{pmatrix}.$$

with $J_{33} = -\beta_{\pi_e^e}$, so that a negative eigenvalue is added to already stable 2-D subsystem. Therefore the steady state of the expectations-augmented real-financial interaction process is, again, locally stable. Hence, given the continuity properties of eigenvalues, the steady state of the dynamics (1)-(2), augmented by the capital gain expectations rule (4), remains locally asymptotically stable even if the weight of chartists in average expectations formation is positive, but is sufficiently small. Intuitively, fundamentalists – if sufficiently dominant – may counteract any destabilising tendencies that chartists may create.

Instead, if the number of chartists, ν_c, the responsiveness of the nominal exchange rate to disequilibria, β_e, and / or the responsiveness of the demand for capital stocks to expected returns, $f'(0)$, are sufficiently high, then one may obtain $J_{33} > 0$ and even $trJ > 0$. In this case, the system becomes unstable by way of Hopf-bifurcations, i.e., in general, by the death of a stable corridor around the steady state or by the birth of stable persistent fluctuations around it. The dynamic system (1), (2), (4) can thus provide a theory of business fluctuations caused by the interaction of real and financial markets.

Note that the previous argument and the existence of Hopf bifurcations is only based on a local analysis. Yet one may expect the presence of chartists to lead to explosive dynamics in general, if the speed of adjustment on financial markets or the responsiveness of the demand for capital stock are sufficiently high. This explosiveness may be tamed far off the steady state if nonlinear changes in behaviour or policy reduce β_e and/or α_k enough to make the system globally stable, thus ensuring that all trajectories remain within an economically meaningful bounded domain. We do not analyse this conjecture further here. Rather, in the next section, we explore the possibility that endogenous changes in the agents' populations, ν_c, reduce the influence of chartists far off the steady state and thereby create turning points in the evolution of capital gain expectations.

3.3 Market expectations dynamics

Even if one rejects the assumption of Rational Expectations, agents in financial markets do learn and they may change their behaviour endogenously in response to changes in the key economic variables. In this section, we adopt a version of the herding and switching mechanism developed by Lux (1995) and Franke (2011), which provides behavioural foundations to the agents' attitudes in the financial market. Unlike in standard DSGE models, we do not start from individual optimisation programmes. The switching mechanism is arguably more realistic than DSGE and it is a very elegant way of capturing *both* rational behaviour *and* purely speculative effects, as well as the phenomenon of herding. In fact, agents decide whether to take a chartist, or a fundamentalist, stance depending on the current status of the economy (captured by the key variables Y, e), on expectations on the evolution of financial gains (π_e^e), and also on the current composition of types of traders in the market (captured by the variable x, defined below).

Formally, suppose that there are $2N$ agents in the economy. Of these, N_c are chartists and N_f are fundamentalists so that $N_c + N_f = 2N$. Let $n = \frac{N_c - N_f}{2}$. Following in particular Franke (2011), we describe the distribution of chartists and fundamentalists in the population by focusing on the difference in the size of the two groups (normalised by N). To be precise, we define

$$x = \frac{n}{N} \in [-1, 1], \quad 1 - x = \frac{N_f}{N}, 1 + x = \frac{N_c}{N}, \tag{5}$$

where, as in Franke (2011), N is assumed to be large enough that the intrinsic noise from different realisations when individual agents apply their random mechanism can be neglected. Formally, given the continuous time setting, the limit of x is taken as N tends to infinity.

Let $p^{f \to c}$ be the transition probability that a fundamentalist becomes a chartist, and likewise for $p^{c \to f}$. The change in x depends on the relative size of each population multiplied by the relevant transition probability.

$$\dot{x} = (1 - x)p^{f \to c} - (1 + x)p^{c \to f}.$$

The key behavioural assumption concerns the determinants of transition probabilities: we suppose that they are determined by a *switching index* s, summarising the expectations of traders on market performance. The switching index depends positively on itself (capturing the idea of herding, see Franke and Westerhoff (2009, p.7), and on economic activity, and negatively on the exchange rate and on average capital gain expectations. Formally, assuming again a functional shape as simple as possible, in order to concentrate on the essential nonlinearities:[6]

$$s \;=\; s_x x + s_y(Y - Y_o) - s_e(e - e_o)^2 - s_{\pi_e^e}(\pi_e^e)^2. \tag{6}$$

This switching index assumes – besides the herding term and the role of economic activity as in Franke (2011) – that the deviations of share prices and capital gain expectations from their steady state values (in both directions) favour opinion making in the direction of the fundamentalists, because doubts concerning the macroeconomic situation become widespread. This change can be interpreted as a change in the state of confidence, whereby agents believe that increasing deviations from the steady state eventually become unsustainable. A similar approach concentrating on price p_k misalignment is used in Franke and Westerhoff (2009, eq.6).

An increase in s is assumed to increase the probability that a fundamentalist becomes a chartist, and to decrease the probability that a fundamentalist becomes a chartist. More precisely, assuming that the relative changes of $p^{c\to f}$ and $p^{f\to c}$ in response to changes in s are linear and symmetric:

$$p^{f\to c} \;=\; \beta\exp(as), \tag{7}$$
$$p^{c\to f} \;=\; \beta\exp(-as). \tag{8}$$

Given the above assumptions, the complete dynamic system becomes:

$$\dot{Y} \;=\; \beta_y((a_y - 1 - a_i i_y)(Y - Y_o) + (a_e - a_i i_e)(e - e_o)) \tag{9}$$
$$\hat{e} \;=\; \beta_e\alpha_e\sigma_e(i^* + \pi_e^e - [i^* + i_y(Y - Y_o) + i_e(e - e_o)]) \tag{10}$$
$$\dot{\pi}_e^e \;=\; \beta_{\pi_e^e}[\frac{1+x}{2}\hat{e} - \pi_e^e] \tag{11}$$
$$\dot{x} \;=\; \beta[(1 - x)\exp(as) - (1 + x)\exp(-as)] \tag{12}$$

As a first step, note that the dynamic system (9)-(12) always has the following steady state: $Y_o, e_o, \pi_{eo}^e = 0, x_o = 0$, where the first three values are uniquely determined (up to flukes), but not x, see below. The asserted uniqueness follows from $\hat{e} = 0$, since we get from this $\pi_{eo}^e = 0$ and on this basis then two equations in the unknowns Y, e which can be solved as intended if policy coefficients are slightly perturbed (if needed).

[6]The details of the approach are in Lux (1995) and Franke (2011).

Equations (9)-(12) represent our baseline DSGD model of a small open economy. All state variables are here dynamic in the sense that their evolution over time is described by more or less gradual adjustment processes, and no algebraic equilibrium condition is involved. Algebraic equations are only used for the equation of the switching index s and the Taylor rule equation of the Central Bank, where the short-term interest rate can be set instantaneously. Both equations do not represent equilibrium conditions. Markets are essentially interconnected and there are various feedback mechanisms present between them.

The key theoretical and policy question is, whether the unfettered market economies described by the DSGD model, where real/financial feedbacks play a prominent role and expectation formation may be affected by herding behaviour, display explosive trajectories, or rather whether they contain some inherent stabilising mechanisms.

If $s_x \leq 1/a$ then this steady state is unique (the first three values are always uniquely determined). If $s_x > 1/a$, then there are two additional steady state values for x_o : the equilibria e_f, e_c, one where chartist are dominant and one where the opposite holds true (all other steady state values remain unchanged). This is suggested by the backward-bending shape of the $\dot{x} = 0$-isocline in figure 4, but is to be obtained in fact by what is shown in figure 3 (which is based on the assumption of given unique steady state values $Y_o, e_o, \pi^e_{eo} = 0$). The figure 3 – and the derivative of this function at 0 – shows that $as_x < 2$ must hold for the case of a uniquely determined steady state value $x_o = 0$.

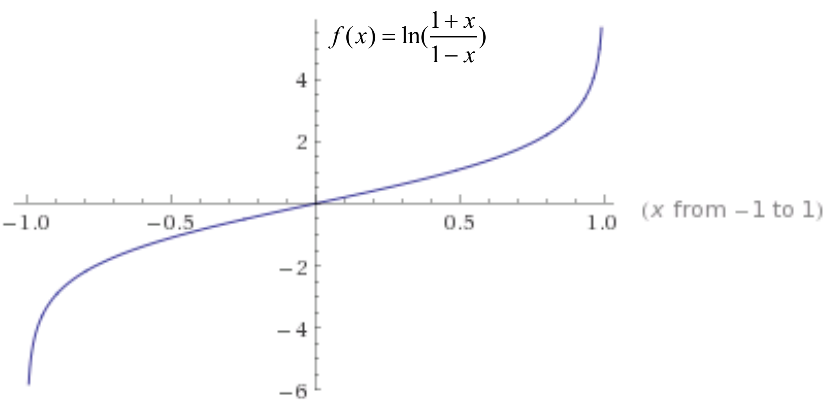

Figure 3: The core of the switching function $f(x) = \ln(\frac{1+x}{1-x})$, $f'(0) = 2$ (for a steady real-financial market configuration).

Before analysing the dynamics of the complete system, it is interesting to consider the properties of the opinion dynamics and the expectational part of the model in isolation. We thus assume that output and dividend payments are fixed at their steady state values. This yields the following 2D system:

$$\dot{\pi}_e^e = \beta_{\pi_e^e}[\frac{1+x}{2}\hat{e} - \pi_e^e], \tag{13}$$

$$\dot{x} = \beta[(1-x)\exp(as(x,\pi_e^e)) - (1+x)\exp(-as(x,\pi_e^e))]. \tag{14}$$

First, note that x always points inwards at the border of the $x-$domain $[-1,1]$. Then, it can be conjectured that there must be an upper and a lower turning point for π_e^e in the economically relevant phase space $[-1,1]\times[-\infty,+\infty]$ and that, if the steady state $(0,0)$ is unstable, the generated cycle stays in a compact subset of this phase space. The expectational herding mechanism would thus be bounded, if taken by itself.

Franke (2011) shows this conjecture to be correct in the context of a formally similar 2D system. Here we simply note that $\dot{x}$ approaches infinity if there is an unlimited increase, or decrease, in the capital gains inflation rate π_e^e. However, as x approaches zero from above or from below, $\dot{x}$ would go to zero if it did not cross the vertical axis at $x=0$. This is a contradiction and therefore there must always be an upper or lower turning point for capital gain inflation or deflation.

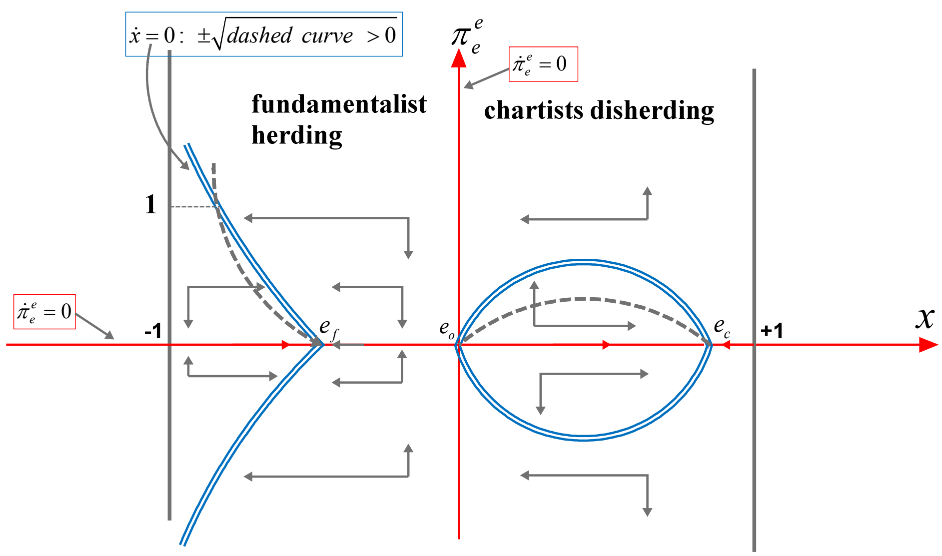

Figure 4: Bounded herding behaviour.

The phase space of the 2D system (13)-(14) is shown in figure 4. The diagram is drawn under the assumption that $s_x > 2/a$, and so there are three steady states (e_f, e_o, e_c). The horizontal axis is an invariant set of the dynamics which cannot be left (or entered) in finite time. Focusing on this part of the $\dot{\pi}_e^e = 0$-isocline we see that both the fundamentalist and the chartist steady state (e_f, e_c) are attracting, but that this only holds for the fundamentalist equilibrium, when the economy is subject to non-zero capital gain expectations.

The $\dot{x} = 0$ isocline is:

$$\pi_e^e = \pm\sqrt{\frac{s_x x - \ln\sqrt{(\frac{1+x}{1-x})/a}}{s_{\pi_e^e}}},$$

and it is attracting with respect to x, since x falls whenever π_e^e is above the isocline and it rises if π_e^e is below it. Note that this isocline is not defined for values of x that make the numerator inside the square root negative. Figure 4 displays some innovative features, as compared to the 2D phase diagrams in the literature, though the outcome of the 2D subdynamics is a fairly trivial one, since only the equilibrium where fundamentalists dominate is by and large a stable one. Figure 4 also suggests that the economy remains in a bounded subset of the state space, if capital gains depart too much from their steady state value (which is zero), due to the strong effects this has on opinion dynamics.

However, because the law of motion of expected capital gains is not easily mapped onto figure 4, it is difficult to analyse the properties of the full 4D system. One should expect the local dynamics to be unstable without policy intervention, since the real-financial markets interaction, in connection with opinion dynamics, is likely to be of centrifugal nature. This raises the issue of the global viability of the unfettered market economy. Based on the analysis of the 2D systems, we cannot conclude that the trajectories of the full 4D dynamic system will always remain in an economically significant subset of the state space.

Given the strong nonlinearity of the opinion part and also in the rate of return function of the 4D dynamics (despite the simple linear behavioral rules we have adopted), we shall address these questions by means of numerical simulations in section 4. They will show that interesting irregular and persistent fluctuations in the real and financial variables of the model can be generated, quite in contrast to what is possible in such a model type under the assumption of the homogeneous rational expectations of the mainstream literature.

3.4 Rational expectations and imperfect exchange rate adjustments

In the case of Rational Expectations one assumes in the deterministic situation, see Turnvosky (1995) for a variety of examples, that π_e^e is simply given by $\hat{e}$. The population dynamics and a separate law of motion for exchange rate expectations is then redundant and we get for the law of motion of the exchange rate in this case:

$$\begin{aligned} \hat{e} &= \beta_e \alpha_e \sigma_k (i^* + \hat{e} - i(\cdot)) \\ &= \frac{\beta_e \alpha_e \sigma_k}{\beta_e \alpha_e \sigma_k - 1}(i_y(Y - Y_o) + i_e(e - e_o)) \end{aligned}$$

We restrict our investigation of the rational expectations approach on the likely case where $\beta_e \alpha_e \sigma_k > 1$ holds true, in which case the fraction to the right is positive (the opposite case is indeterminate in the language of the rational expectations school). Formally seen, the 2D dynamics of section 2 is now self-contained in this modification of its second law of motion.

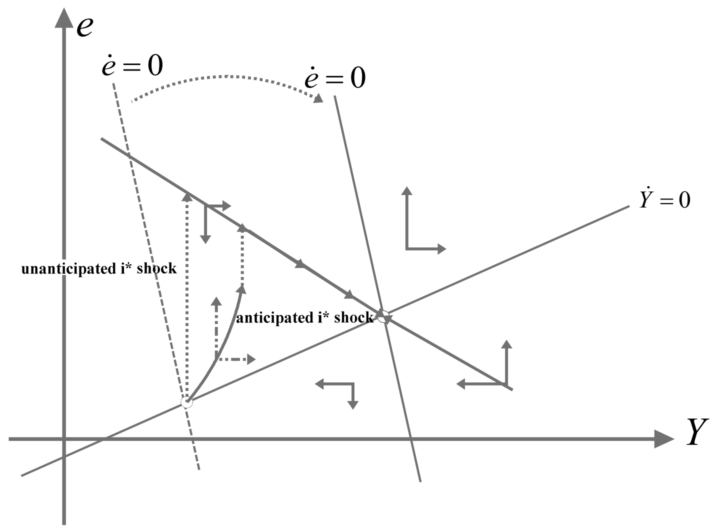

Figure 5: Rational expectations imply real-financial market interactions that – after a tailored jump in the exchange rate e – converge to the steady state of the post-shock dynamics.

Moreover, the isoclines in figure 1 remain in their position and the dynamics are now pointing upwards above the $\dot{e} = 0-$isocline and vice versa. This situation is shown in figure 5. In this figure we also consider a shock to the economy which moves both isoclines into a new position (for graphical reasons). Assuming the economy to have been in the steady state of the old dynamics then implies (by assumption) a jump of the exchange rate share prices e onto a unique position on the stable arm of the post-shock dynamics (if these dynamics are determinate) along which they and output then converge to the new steady state position. This is a very tranquil theoretical scenario for what is assumed to happen in the FX markets, as illustrated by Fig. 5.

4 Numerical simulations

Turning to the whole $4D$ system with an endogenously determined amount of behavioral traders again, we use numerical means now to gain more detailed insights to its dynamical behavior. In order to run the numerical simulations the software package *E&FChaos* by Diks et al. (2008a) has been applied.[7] It allows for the implementation of dynamic economic models in a very user-friendly plain text file and offers routines for many tasks required for performing analyses of dynamical systems. The code for the model at hand looks as follows:

[7]The software can be downloaded from
http://www1.fee.uva.nl/cendef/whoiswho/makeHP/page.asp?iID=19

```
Y=1.02, e=2, pie=0, x=0
bpi=1, be=2.5, alpe=0.25, sige=4, beta=1.4, sx=1.2, spie=0.5, a=1, by=2, ay=0.6,
ai=0.1, ae=0.35, iy=0.1, ie=0.1, sy=0.2, se=0.5, Yo=1, eo=2, stari=0.05

c 0.01
i=stari+iy*(Y-Yo)+ie*(e-eo)
Y1=by*((ay-1)*(Y-Yo)-ai*(i-stari)+ae*(e-eo))
e1=e*be*alpe*sige*(stari+pie-i)
pie1=bpi*((1+x)/2*be*alpe*sige*(stari+pie-i)-pie)
x1=beta*((1-x)*math.exp(a*(sx*x+sy*(Y-Yo)-se*(e-eo)^2-spie*pie^2))
              -(1+x)*math.exp(-a*(sx*x+sy*(Y-Yo)-se*(e-eo)^2-spie*pie^2)))

Y=Y1
e=e1
pie=pie1
x=x1
```

The line c 0.01 declares that the Runge-Kutta procedure is used to run the model as a continuous-time model with step size 1/100. The preceding lines provide the initial conditions and the parameter values for the simulation (each without hard return). Next, there is the definition of the employed Taylor rule. The next four equations provide the four laws of motion of the model and the subsequent equations provide the updating procedure for the state variables of the model. As a starting configuration of the model, reasonable values have been assigned to the parameters and initial conditions. For the time being they are neither calibrated, nor estimated, since the expectational part is not observable as such and empirical validation is beyond the scope of the paper, but remains a challenging task for future research.

In the first simulation, we indicate where the 3D subdynamics – resulting when population shares are held constant ($\dot{x} = 0$) – loses their stability (at around $\beta_\pi = 1.5$) in figure 6 by way of a Hopf bifurcation. The bifurcation seems to be of a degenerate type, i.e., no limit cycle is born or lost at this bifurcation point. The figure primarily provides the point of departure for the study of the opinion dynamics that follows.

The opinion dynamics switched on again in figure 7 as indicated by the parameter values shown above. It reveals the surprising result that it is relatively complex around $\beta_\pi = 0.64$, but converges before and after this area, beyond the original bifurcation point.[8] This is by and large confirmed by the Largest Liapunov Exponent shown in figure 8 for the same parameter range. The range for irregular orbits is around 0.64 while we have a simple limit cycle shortly before it (figure 9, $\beta_\pi = 0.62$). Such limit cycles can become irregular too when

[8]We show ten years of iteration after transient period of 300 years.

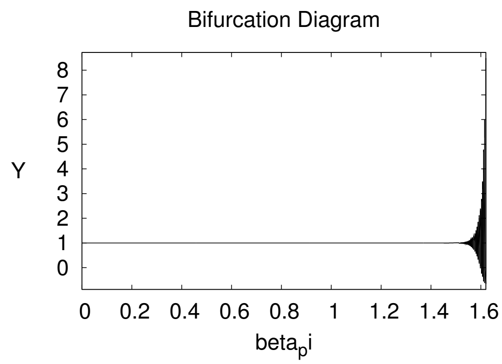

Figure 6: A Hopf bifurcation which leads into explosive instability

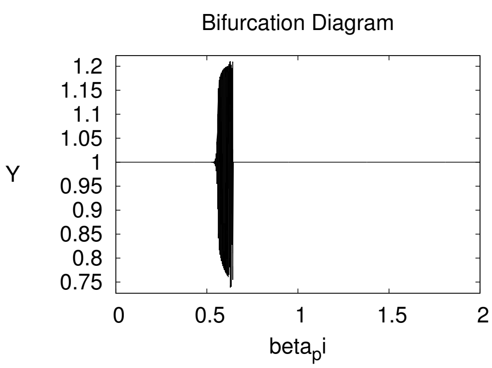

Figure 7: Stabilizing opinion dynamics

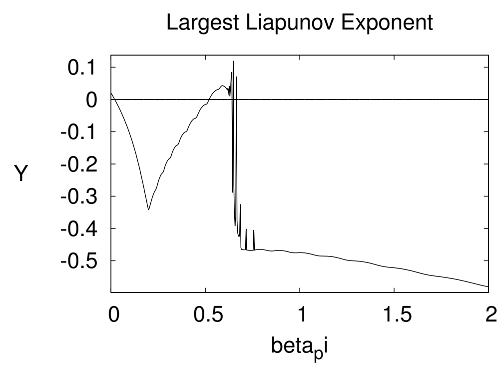

Figure 8: Are there 'chaotic' trajectories?

noise is added to them (here the noise is added uniformly, with a variance of 0.5% of output levels). After a while, the astonishing thing is that the deterministic part is reduced in its extent through the addition of noise (figure 10).

Figure 11 demonstrates – in correspondence to figures 9 and 10 – that the time series can become irregular in the purely deterministic core of the model which is based on a continuous ODE system of dimension 4. This is only possible with at least three laws of motion, as we know from the famous Rössler dynamics (see the manual which is accompanying the software we are using (Diks et al. 2008b)). Adding again noise to the deterministic part of the model provides the same outcome as we have observed in the simple limit cycle case (figure 12).

Independent of policy to be discussed in the next section, the considered dynamics is very robust (viable) over large ranges of parameter values. We show this in the next two bifurcation diagrams with respect to the parameters β_x and β_e and for the share of chartists in the population of risk traders.

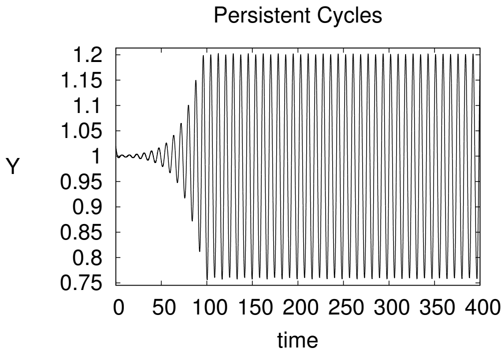

Figure 9: A limit cycle result

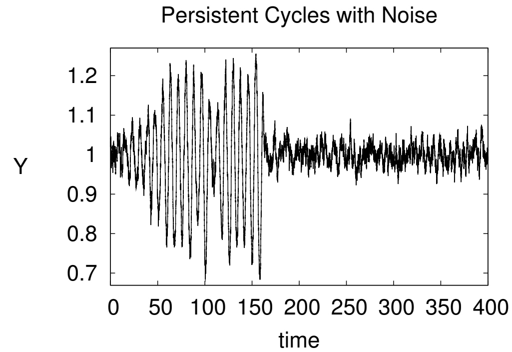

Figure 10: Adding noise to the limit cycle

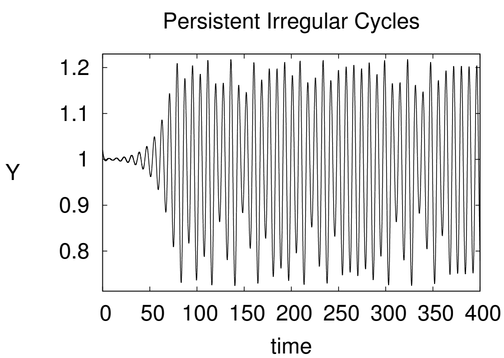

Figure 11: Smooth dynamics with irregular business fluctuations

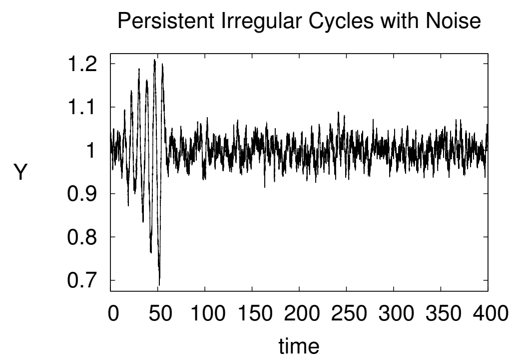

Figure 12: Adding noise to the already irregular output dynamics

In the first case, the asymptotic stability of the centre equilibrium $x = 0$ gets lost after a while and gives rise to larger fluctuations in population shares which are tamed at a higher speed of adjustment of the population of chartists by the asymptotic stability of a population equilibrium where fundamentalists outweigh chartists. In the second figure, we start from a stable chartist equilibrium which in the middle of the parameter range loses its stability through large fluctuations in population dynamics and falls into a fundamentalists equilibrium of nearly equal size (viewed from its absolute value). The astonishing thing again is – though both parameters can be expected to destabilise the economy – that it returns to asymptotic stability after a parameter range which is characterized by significant turbulence.

We close our simulation runs with two studies (figures 15 and 16) which show that the generally unstable chartist equilibrium can be persistent for quite a while (with and without noise), but will eventually be surrounded by explosive motion and then collapse to a stable equilibrium of fundamentalist type. This may also happen in the preceding figures when the

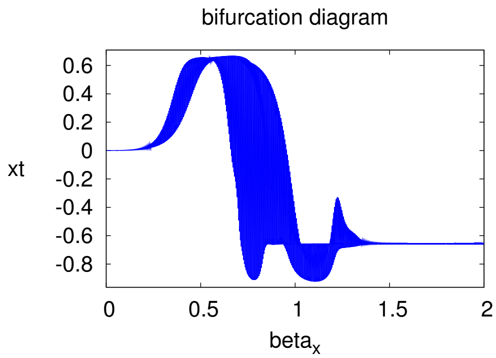

Figure 13: Robustness test of the dynamics for the adjustment speed of the chartists; population

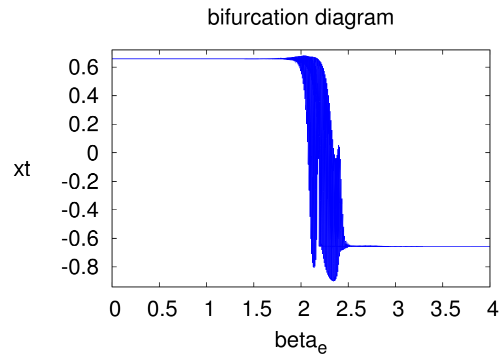

Figure 14: Robustness test of the dynamics for the adjustment speed of the exchange rate

time period for the simulation is made considerably longer.

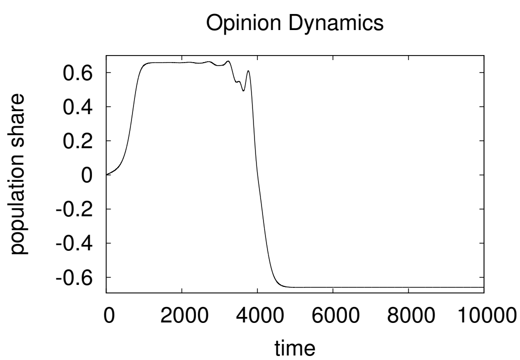

Figure 15: Prolonged, though only temporary dominance of chartists' positions

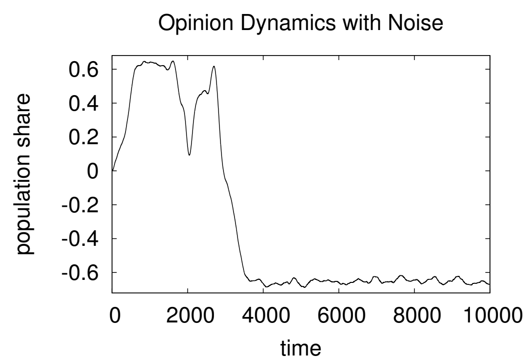

Figure 16: Adding noise to the situation shown in figure 15

5 Policy Options

The key theoretical, empirical, and policy question is whether unregulated market economies contain some mechanisms which ensure the stability of equilibria, or rather centrifugal forces prevail, making the equilibrium unstable and, potentially, the system non-viable. Numerical simulations show that global stability can be ensured if, far off the steady state, opinion dynamics induces fundamentalist behaviour during booms and busts which ensures that there are turning points in both of these situations. However, both the local analysis and the simulations suggest that market economies can be plagued by fluctuations and recurrent crisis phenomena. That is the reason why at least a quick review of possible policy measures is indispensable.

We show that policy measures often advocated in the Keynesian literature, namely Tobin taxes, here on capital gains in the FX market, and countercyclical fiscal policy can mitigate those problems. Moreover, monetary policy which follows a FX market-oriented Taylor rule or is strongly output-oriented can enforce stability.

The model highlights *two* sources of instability in the economy: the interconnection between real and (foreign) financial markets, and the destabilising role of chartists in asset markets.

First of all, as many authors since Keynes (1936) and Minsky (1982, 1986) have stressed, the main function of financial markets should be to ensure the efficient allocation of savings, and gambling activities should be constrained. It is therefore appropriate to consider a *Tobin type tax (or subsidy)* on capital gains in the FX market at rate τ_e, such that total tax revenue is equal to[9]:

$$\tau_e \alpha_e \sigma_e(\cdot)$$

Therefore the law of motion for capital gain expectations (11) can be re-written as:

$$\dot{\pi}_e^e = \beta_{\pi_e^e}[\frac{1+x}{2}(1-\tau_e)\hat{e}_e - \pi_e^e],$$

and Tobin taxes indeed have a stabilising effect by weakening the impact of chartists on the process of market expectation formation.

Additional numerical analysis shows that capital gains taxation might ensure stability if conducted with sufficiently high rates. The bifurcation diagram points to a rate starting with not less than 20 percent for the parameter configuration chosen before.

Conventional countercyclical fiscal policy could also reduce fluctuations if it acts on the propensity to spend parameter a_y in the equation of the dynamic multiplier.(To be seen in figure 18.) But the prerequisite here is rather demanding as values up to 0.75 are associated with increasing explosiveness. Only after passing this threshold value is convergence achieved.

Turning to monetary policy, the Taylor rule $i = i^* + i_y(Y - Y_o) + i_e(e - e_o)$ works as a built-in stabilizer, which eliminates an accelerating feedback structure between e and Y and, as already seen, makes the interaction of real and foreign bond markets in isolation a stable one. In order to assess the effects in combination with opinion dynamics, we have to rely on numerical means again. The figure 19 shows the bifurcation diagram for the policy

[9]Tobin (1978, 1996) proposed a currency transaction tax, which might work as a stabilising device for international financial markets and generate revenues to be used by an international authority to finance economic development.

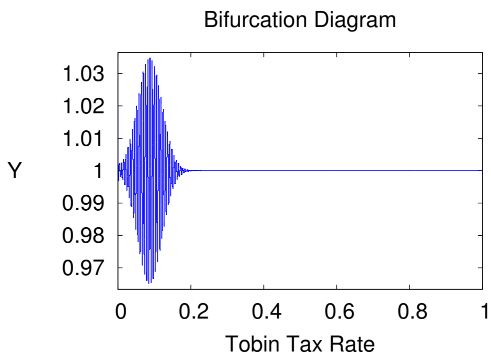

Figure 17: Capital gains taxation

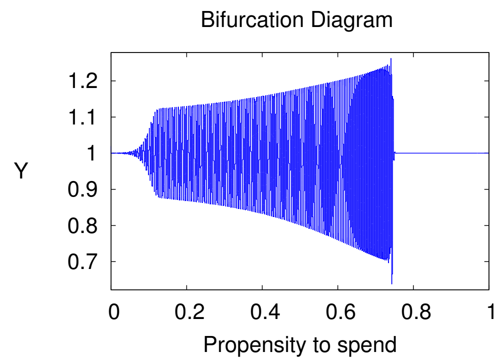

Figure 18: Countercyclical Fiscal Policy

parameter i_e, the positive reaction of the interest rate with respect to currency devaluations (and its decrease in the case of appreciations). Up to the critical area already studied, this policy reaction always provides convergent dynamics and is therefore supportive (also in areas where there are limit cycle fluctuations). An orientation towards output targeting can work successfully as well, but it needs to be conducted with sufficient strength. As it is clear from figure 20, convergent dynamics can only be achieved for rather high values of the output reaction parameter i_y in relation to FX-targeting.

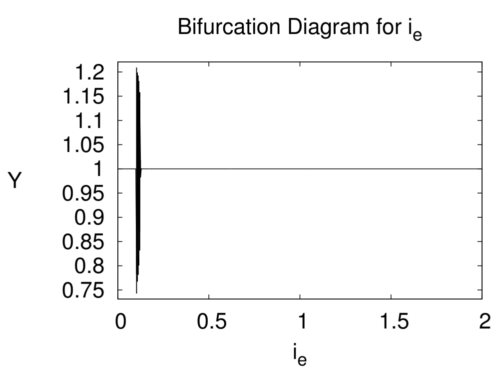

Figure 19: The impact of FX-oriented interest rate policy over the range $i_e \in [0, 2]$

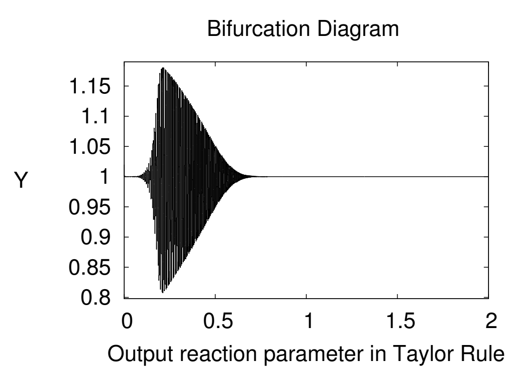

Figure 20: Output-oriented interest rate policy over the range $i_e \in [0, 2]$

6 Concluding Remarks

In this paper a stylized model of a small-open economy with endogenous market expectations formation based on heterogeneous behavioral FX expectations has been presented. The assumptions of market clearing and rational expectations were dropped, and instead, a set of gradual, dynamic adjustment processes taking place on highly interconnected real and financial markets was assumed. The resulting framework – where foreign exchange markets influence the state of confidence of the economy and thus consumption and investment decisions, which in turn influneces the performance of the real sector – has been shown to be tractable enough to be – at least partially – investigated by means of analytical tools on the one hand, but on the other hand elaborate enough to generate complex dynamics.

A central element of the analyzed theoretical framework was the assumption of FX markets populated by chartists and fundamentalists. One may argue that the theoretical expectation formation rules that characterise chartists and fundamentalists, should be replaced by more sophisticated backward- and forward-looking rules based on econometric estimation techniques. It would certainly be interesting to analyse the impact of different expectation formation rules on the system. We do not, however, expect these changes to significantly affect the key conclusions of our analysis. Secondly, in our formalisation of market expectations, we supposed that the agents' guessing process is stopped after one step: market expectations are what agents think they will be on average. We considered this as a first step into the analysis of more complex processes of aggregate expectation formation. Once one drops the assumption of Rational Expectations, many further possibilities can be explored, including Keynes' (1936) celebrated "third degree" process, where agents try to anticipate what *average opinion* expects average opinion to be. We leave this suggestion for further research.

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