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On the Effectiveness of Effluent Charges

by

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1. Introduction

For most readers it may go without saying that an effluent charge levied on a polluting firm tends to be an effective means of controlling pollution. If, nevertheless, we seek in this article to prove this point, it is because the decision problem of a producer, subject to such a tax, typically lacks the nice property of conjugate pairs that would allow a quick, unambiguous answer[1]. Consequently, there have been some doubts raised in the literature. More precisely, we intend to demonstrate within a partial equilibrium framework that levying an effluent charge on either a monopolist or on perfectly competitive firms is an effective environmental policy both in the short-run and in the long-run. In order to show this, one has, however, to make a slightly more intensive use of the rich implications of the optimization hypothesis than is usually required in the analysis of indirect taxes.

There is a vast body of literature dealing with effluent charges within a partial equilibrium setting. For instance, using a rather restrictive assumption about the feasible technology, SIEBERT [1976] has shown in this Journal that imposing an effluent charge on a polluting monopolist will unambiguously lower its pollution. As Siebert has already pointed out, this result applies also (in the short-run) to the case of a perfectly competitive producer. On the basis of his assumptions, it could even be shown that the effluent charge induces simultaneously a reduction in output and an increase in abatement efforts. As will become apparent later, these last two specific conclusions do not necessarily apply if a less restrictive assumption with respect to technology is made, but the main conclusion does. According to this, the tax is effective in lowering pollution, in the short-run. The long-run consequences of an effluent charge, in contrast, have been less intensively explored. The most comprehensive treatment still appears to be that of BAUMOL and OATES [1975] who dedicate nearly an entire chapter of their well-known textbook to this question without, however, reaching a conclusive answer[2]. If an unambiguous answer is presented

[1] On the usefulness of the conjugate pairs property see SAMUELSON [1947] or, with a different view, ARCHIBALD [1965].

[2] See BAUMOL and OATES [1975], Chapter 12 and therein particularly Proposition 4.

here, however, it is not due to reliance on more restrictive assumptions. In fact, in what follows use will be made of all the assumptions and even the notation of Baumol and Oates. The explanation is simply that these authors base their analysis of the long-run on a defective approach[3] that prevents them from reaching a clear-cut conclusion.

The layout of this paper is as follows: after a reiteration of the basic assumptions, the comparative statics that determine the reactions of both the monopolist and the perfect competitor will be briefly developed. Since the monopolist can be treated formally in practically the same manner as the perfect competitor in the short-run, both will consequently be analyzed jointly. The following section then explores the long-run effects of an effluent charge on the level of pollution of an entire industry in long-run competitive equilibrium. Finally, a brief summary follows.

2. *The Assumptions and the Associated Comparative Statics*

Consider a firm which produces a marketable output y jointly with an unwarranted product s that damages the environment and is levied with an effluent charge. Following Baumol and Oates ([1975], Chapter 12) let

$$\pi = yp(y) - c(y, a) - ts(y, a) \tag{1}$$

be the profit function of the individual producer where p denotes the price of the output, c denotes total cost of production, including the cost of abating emissions, a is the level of abatement efforts and t is the effluent charge. In order to maintain as high a degree of generality as possible within this framework, we will merely require that the technology that is implicit in the cost function $c(y, a)$ and the emissions production function $s(y, a)$ be compatible with profit maximizing behaviour. To rule out corner solutions and other unwieldy cases that go along with nonconvexities, however, we will assume that the technology always permits a regular interior maximum. Thus, in the profit maximum (marked here by an asterisk) both the first-order conditions[4]

$$\pi_y^* = p + yp_y - c_y - ts_y = 0 \ , \tag{2.1}$$

[3] In their analysis of the long-run equilibrium of a competitive industry Baumol and Oates rely, as is commonly done, on the zero-profit condition. Yet in the profit function of an individual firm they replace the product price by the demand function for the entire industry; *op. cit.*, eq. (14) on p. 186. In doing so, they implicitly equate the output of a single firm to that of the entire industry and, what is more important, the long-run change in an individual firm's output to the long-run change in the industry's output. This is, of course, not feasible.

[4] To simplify notation, partial derivatives will be indicated by subscripts, i.e. $\pi_y = \partial\pi/\partial y$ etc.

(2.2) $$\pi_a^* = -c_a - ts_a = 0$$

and the second-order sufficient conditions hold. The latter means that the second differential of the profit function is strictly negative for all non-trivial variations (dy, da) in output or abatement activities or, what amounts to the same, that the associated Hessian matrix

(3) $$H^* = \begin{bmatrix} \pi_{yy}^* & \pi_{ya}^* \\ \pi_{ya}^* & \pi_{aa}^* \end{bmatrix}$$

be negative definite. As is well known, this in turn requires

(4) $$\pi_{yy}^* < 0, \quad (\pi_{aa}^* < 0), \quad |H^*| = \pi_{yy}^* \pi_{aa}^* - \pi_{ya}^{*2} > 0$$

and implies in particular that the inverse

(5) $$H^{*-1} = \frac{1}{|H^*|} \begin{bmatrix} \pi_{aa}^* & -\pi_{ya}^* \\ -\pi_{ya}^* & \pi_{yy}^* \end{bmatrix}$$

exists and is negative definite.

For the profit maximizing competitive firm which views the price p as given, we have then the explicit comparative static system

(6) $$\begin{bmatrix} y_p^* & y_t^* \\ a_p^* & a_t^* \end{bmatrix} = H^{*-1} \begin{bmatrix} -1 & s_y^* \\ 0 & s_a^* \end{bmatrix}.$$

Note that the comparative static system associated with a monopolist appears to be formally exactly alike that associated with a perfect competitor, except that it lacks the first column in the reactions matrix on the LHS and likewise the first column of the second matrix on the RHS of (6). This is simply a consequence of the fact that the monopolist does not of course conceive of the product price as a given parameter. Also, there is no great difference in substance between the perfect competitor and the monopolist because the feasible technology that is compatible with a regular profit maximum and hence with (2) and the negative definiteness of (3) or (5) is only a little more restricted in case of perfect competition. But even in the latter case, the model of Baumol and Oates is capable of dealing with a fairly wide range of feasible technologies[5].

[5] A referee has pointed out that this is still not all that general, because the Baumol and Oates framework used here cannot accomodate the more general joint production technology considered, for instance, in Pethig [1979] or Gronych [1980]. While this is undoubtedly true we would like to mention, however, that all our results below carry over equally well to that case if we were considering instead profit-maximizing producers whose technological (and other) constraints can be described by the (possibly vector-valued) function $F(y, s, x) = 0$, in which y and s have the same meaning as before and x is some vector of inputs including abatement activities.

Note that the first-order conditions say little about how the first partial derivatives of the cost function $c(y, a)$ or the emissions production function $s(y, a)$ should look. It would appear natural, however, to assume that

$$(7) \qquad c_y > 0, \quad c_a > 0, \quad s_y > 0 \quad \text{and} \quad s_a < 0$$

holds, i.e. that production costs rise with increasing output and increased abatement efforts, and emission volume rises with increased output and falls with larger abatement efforts[6]. The second-order conditions also tell little about the second partial derivatives of c and s, and in particular they tell absolutely nothing about the signs of the mixed partials. Knowledge of these signs would be quite helfpul in the comparative static analysis but since there appears to be no sound a-priori-reasoning on the basis of which they could be determined, we will abstain from proposing or using any specific assumptions in this respect.

The assumptions mentioned so far suffice to analyze the behaviour of a monopolist or the short-run reactions of a perfectly competitive firm. However, in order to determine the long-run reactions of an entire (perfectly competitive) industry, two further assumptions are needed. The first is that aggregate demand for the industry's aggregate output Y is downward sloping, i.e. $p = p(Y)$ with $dp/dY \leqq 0$. This assumption may look inconspicuous, but it is not. It rules out two cases which are considered in the literature as possibly causing an anomalous relationship between an effluent charge and aggregate pollution levels[7]. The second assumption relates to the entry and exit-mechanism. In this context we will again simply follow Baumol and Oates and assume that the long-run equilibrium of the perfectly competitive industry is characterized by zero profits.

3. *The Perfectly Competitive Firm in the Short-run and the Monopolist*

We define the short-run to be long enough to allow the individual firm to adjust to an exogeneous change but too short for the market to react in turn. Thus, the short-run effects of a change in the effluent charge on the behaviour of the individual firm are those given in the second column of (6). Despite the simple structure of the problem, neither the short-run reaction of output, y_t^*,

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6 Clearly, the derivative s_a cannot be positive. If it were, we would be using a sign-reversed definition of what constitutes abatement.

7 We will see below that if aggregate demand were upward sloping, our long-run analysis would cease to be unambiguous. See PETHIG [1979], pp. 132ff. As a referee has pointed out, the other case is less obvious by writing simply $p = p(Y)$ and thus not allowing the level of emissions to enter the demand function explicitly, we are ruling out a possible feedback effect that could well reverse our results. For an analysis of this situation see again PETHIG [1979] or GRONYCH [1980], pp. 85ff.

nor the short-run reaction in abatement efforts, a_t^*, can be uniquely determined. And this remains true even if we were to use the reasonable assumptions set out in (7). In view of (5) and (4), the unknown sign of the cross partial derivative, π_{ya}^*, is clearly the reason for this ambiguity[8]. The interesting aspect of this negative result is that a rise in effluent charges need not necessarily induce a simultaneous reduction in output and intensified abatement efforts as one might be inclined to expect.

While it is obviously impossible to qualitatively determine in general the specific adjustments in output and abatement efforts, this ambiguity disappears when it comes to determining the short-run change in the level of emissions. Differentiating $s(y^*, a^*)$ with respect to t we have, using (6),

$$s_t^* = (s_y^*, s_a^*)\,(y_t^*, a_t^*)' = (s_y^*, s_a^*)\,H^{*-1}\,(s_y^*, s_a^*)' \leqq 0 \tag{8}$$

where the sign follows unambiguously from the fact that H^{*-1} has to be negative definite. Ruling out the trivial case $s_y = s_a = 0$, i.e. the case where the emission level is independent of both output and abatement efforts, we even have the strong inequality sign in (8). There is consequently no doubt that in the short-run the emissions of the individual competitive firm and hence also those of the entire industry will fall when the effluent charge is raised.

Since the comparative statics for the monopolist are virtually the same as those for the competitive firm just considered, the results derived in this section apply without amendment to the monopolist. It can thus be held with certainty that, in response to an increase in the effluent charge, the emissions of the monopolist will fall; whether the monopolist will step up or lower his output or his abatement efforts cannot be said. This latter ambiguity gives rise to an interesting possibility: since an increase in output by the monopolist cannot be ruled out, and since this short-run reaction represents at the same time his final response, it may well be that as a result of an increase in the effluent charge the market actually ends up with an improved supply of the monopolist's commodity. As will be seen in the following section, such a constellation cannot possibly arise as a long-run solution if the commodity were supplied competitively.

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[8] If we were to assume $\pi_{ya}^* = 0$, however, the Hessian matrix (3) and its inverse (5) would be diagonal. Using (4) through (7), we would then find immediately $y_t^* < 0$ and $a_t^* > 0$, i.e. a fall in output and a rise in abatement efforts as the short-run reaction to an increase in effluent charges. Even if it may not be entirely obvious, this setup corresponds exactly to the one analyzed by SIEBERT [1976]. Substituting all his technological equations into his profit function it can be easily seen that the associated Hessian matrix is likewise diagonal. The question remains whether the underlying assumptions are reasonable. They require that the marginal productivity of abatement efforts be independent of the output level and that conversely the marginal cost of production also be independent of the level of abatement activities. In general, one may not wish to rule out such cross effects even though it is undoubtedly difficult to form convincing hypotheses about their specific nature.

4. The Long-run Competitive Equilibrium

Since any change in an exogeneous variable such as the product price p or the effluent charge t will affect the competitive firm's maximum profit π^*, it is obvious that a change in the effluent charge requires in the long-run a compensatory adjustment in the output price if the initial zero-profit equilibrium is to be restored. Thus, $d\pi^* = \pi_p^* dp + \pi_t{}^* dt = 0$ must hold in the long-run. Using the envelope theorem which states $\pi_p^* = y^*$ and $\pi_t^* = -s^*$ we therefore conclude that

$$dp = \frac{s^*}{y^*}\, dt \; . \tag{9}$$

This means that to restore long-run equilibrium the product price will have to change s^*/y^*-times the change in the effluent charge. What induces this adjustment, how long the process would take and, in particular, whether it would be stable or not[9] can obviously not be answered without further specific assumptions. But since the analysis of such dynamic aspects does not appear to promise rewarding insights this issue will basically be left to one side.

Armed then with the knowledge that the long-run situation is characterized by the parameter changes $(dt \neq 0,\ dp = (s^*/y^*)\, dt)$, we can now investigate the long-run effects of the effluent charge. Considering first the reactions of the individual competitive firm which reaches the new industry equilibrium, we find from (6) and (9) that

$$\begin{bmatrix} dy^*/dt \\ da^*/dt \end{bmatrix} = \begin{bmatrix} y_t^* \\ a_t^* \end{bmatrix} + \begin{bmatrix} y_p^* \\ a_p^* \end{bmatrix} \frac{dp}{dt} = H^{*-1} \begin{bmatrix} s_y^* - s^*/y^* \\ s_a^* \end{bmatrix} . \tag{10}$$

Again, as in the analysis of the short-run effects, it is impossible to determine qualitatively the individual firm's response in output and abatement efforts to the effluent charge. This is certainly not surprising, since these long-run reactions consist now, as (10) indicates, of two possibly contradictory reactions, namely the immediate reaction to the change in the charge and the reaction to the consequent change in the product price. For the long-run change in the firm's emission level we then have, using (10),

$$ds^*/dt = (s_y^*, s_a^*)\,(dy^*/dt, da^*/dt)' = (s_y^*, s_a^*)\, H^{*-1}\,(s_y^* - s^*/y^*, s_a^*)' \tag{11}$$

to which in contrast to the short-run result (8) no sign can be given, since the RHS does not reduce to a quadratic form in H^{*-1}. Consequently, the result

[9] There can be little doubt that the assumption of a strictly downward sloping market demand curve, together with the standard assumption of profits regulating entry and exit of firms in the industry, suffices to assure at least local stability of the long-run equilibrium price.

might well be that the surviving firms will eventually pollute more than they did before the effluent charge was raised.

This ambiguity has, however, no consequences for the industry's aggregate long-run level of pollution. An unambiguous sign in (11) would be of little help anyway in determining this, because it is not the change in a surviving firm's absolute level of pollution which is decisive but the change in its pollution *intensity*. And in this respect, the answer is clear-cut: differentiating the intensity of pollution per unit of output, s^*/y^*, totally with respect to t and using (10) and (11) we have

$$(12)\qquad \frac{d}{dt}\left(\frac{s^*}{y^*}\right)=\frac{1}{y^*}\left(\frac{ds^*}{dt}-\frac{s^*}{y^*}\frac{dy^*}{dt}\right)= \frac{1}{y^*}(s_y^*-s^*/y^*,s_a^*)\,H^{*-1}(s_y^*-s^*/y^*,s_a^*)'\leqq 0\ .$$

Thus, disregarding the trivial constellation where the RHS vanishes[10], we can be sure that pollution per unit of output will definitely fall in the long-run in response to an increased effluent charge[11]. Now, since in view of (9) and the downward-sloping market demand curve aggregate output cannot have risen, and since the industry's aggregate level of pollution is nothing but the product of this aggregate output and the pollution intensity, we can immediately conclude that the industry's long-run level of pollution will fall in response to a rising effluent charge.

The importance of the assumption of a downward-sloping market demand curve for the conclusion just reached is quite evident at this point. If we were instead to permit the demand curve to have upward-sloping segments, the sign in (12) would, of course, remain unaffected und thus pollution intensity would still have to decline in the long-run. But the conclusion with regard to the long-run change in aggregate levels of pollution would be endangered, since a rise in the effluent charge and the consequent increase in the output price would then be compatible with a long-run increase in aggregate output and thus pol-

[10] Note that the RHS vanishes only if both $s_a^*=0$ and $s^*=s_y^*y^*$ holds, i.e. if there is no abatement technology available and if simultaneously pollution is simply proportional to output. Both constellations are not all that likely but some attention has been paid to them in the literature. See the literature quoted in BAUMOL and OATES [1975], Chapter 12.

[11] Since the sign came out so nicely in (12) one might be inclined to hope that it should be possible to determine along similar lines how long-run pollution per unit of output will develop, if, instead of taxing pollution, a subsidy is paid if actual pollution s^* remains below a certain target value $\bar{s}$. Assuming that the subsidy amounts to $t(\bar{s}-s^*)$ where t denotes then the (constant) subsidy rate, it could be shown that $d((s^*-\bar{s})/y^*)/dt\leqq 0$ must hold in the long-run. But this is not sufficient to determine the only relevant sign of $d(s^*/y^*)/dt$. Baumol and Oates' conjecture that a subsidy is likely to be a counterproductive measure in the long-run therefore cannot be made more precise.

lution. It is, however, difficult to envisage how such a constellation could possibly constitute a stable long-run equilibrium.

Returning then to our assumptions, another aspect of the central conclusion just reached should be fairly obvious. Not only the technology but also the market conditions are basic to the success of an effluent charge in the long-run. While it is true that the technology available to the individual firm determines how much pollution intensity will decline and even how much the output price must rise in the long-run, it is clearly the market which decides on the industry's long-run output and consequently its pollution as well. The more difficult it is for the industry to pass on the effluent charge to its output price in the sense of lost sales volume, the more effective the effluent charge will be in fighting pollution[12].

Summary

Within a partial equilibrium framework and based on fairly general assumptions, it is demonstrated that an effluent charge is indeed an effective means of controlling aggregate pollution if the polluter is either a monopolist or a competitive industry. Part of this result is the intermediate finding that in the case of a competitively organized industry, the long-run volume of pollution per unit of output will almost certainly decline and at worst stay constant if the charge is raised. Yet to those living close to competitive firms, these results may offer little consolation. Since adjustment to the new long-run equilibrium may conceivably go along with a heavy concentration of production within a few remaining firms, each of these survivors may in fact eventually pollute more than it did before the effluent charge was raised.

Zusammenfassung

Die Wirksamkeit von Emissionssteuern

Auf der Grundlage eines von Baumol und Oates verwendeten partialanalytischen Ansatzes und entgegen von diesen Autoren geäußerten Zweifeln wird gezeigt, daß Emissionssteuern sowohl kurzfristig als auch langfristig ein eindeutig wirksames Mittel zur Bekämpfung des im Rahmen einer konkurrenzwirtschaftlich oder monopolistisch organisierten Produktion anfallenden Schadstoffausstoßes darstellen. Als Zwischenergebnis zeigt sich dabei, daß im

[12] As indicated earlier, our results continue to hold if we allow for a somewhat more general technology than the Baumol and Oates model does. In particular, we could permit the effluent charge to be passed back to the suppliers of inputs without affecting the results. In analogy to the assumption about market demand, however, it would have to be assumed that the factor supply curves for the industry are in turn upward-sloping.

Falle vollständiger Konkurrenz der Schadstoffausstoß pro Outputeinheit im Gefolge einer Steueranhebung langfristig sinken muß. Dennoch reichen diese Ergebnisse nicht aus um zu sichern, daß auch der individuelle Schadstoffausstoß der in der Branche verbleibenden Produzenten nach einer Steueranhebung langfristig abnimmt.

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